Starter Activity – Where is the Flaw in the Logic?

\[ \begin{align*}
    a &= b \\
    a^2 &= ab \\
    a^2 - b^2 &= ab - b^2 \\
    (a - b)(a + b) &= b(a - b) \\
    a + b &= b \\
    \text{Since } a &= b, b + b &= b \\
    2b &= b \\
    \therefore 2 &= 1
\end{align*} \]
Prove the following

1. If $x$ and $y$ are both odd, prove that the product of the two is also odd.

2. For any odd integer $x$, prove that $x^3$ is odd.

3. Prove that if $n$ is a positive integer then $3^{2n} - 1$ is divisible by 8.

(hint: multiply out $(3^n-1)(3^n+1)$)
Trigonometric Identities

Prove that \( \sin^2 A + \cos^2 A = 1 \)
Forms of Proof

- Proof by Direct Argument
- Proof by Contradiction
- Proof by Exhaustion
- Proof by Induction
An Example of Proof by Contradiction

- Prove that $\sqrt{2}$ is irrational

Let us suppose that it is rational, i.e. that $\sqrt{2} = \frac{m}{n}$, where $m$ and $n$ have no common factors.

$$2 = \frac{m^2}{n^2} \text{, so } 2n^2 = m^2$$

$m^2$ is even, so $m$ must also be even i.e. $m = 2p$

$$2n^2 = (2p)^2 = 4p^2$$

$n^2 = 2p^2$ , so $n$ must also be even

However, this violates the assertion that $m$ and $n$ have no common factors, as they have a common factor of 2.

Therefore $\sqrt{2}$ is not rational i.e. irrational.
Try and apply that to the following problem!

Prove that $\log_2 3$ is irrational.