Starter Activity – Where is the Flaw in the Logic?

- $$a = b$$
  $$a^2 = ab$$
  $$a^2 - b^2 = ab - b^2$$
  $$(a-b)(a+b) = b(a-b)$$
  $$a + b = b$$
  $$\text{Since } a = b,\ b + b = b$$
  $$2b = b$$
  $$\therefore 2 = 1$$

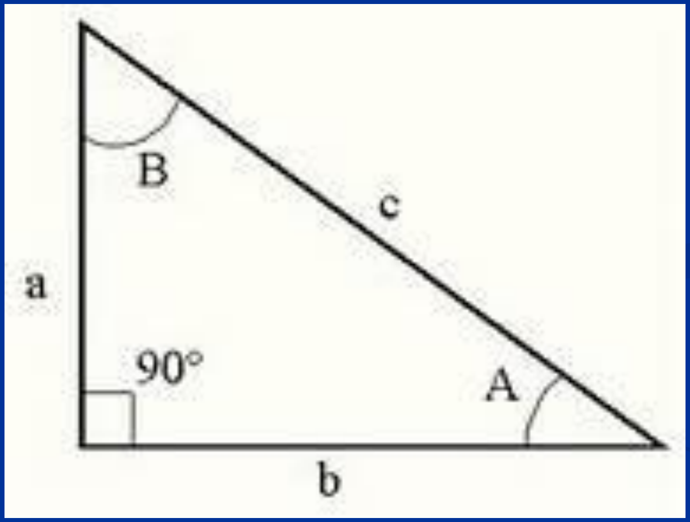
Prove the following

1) If x and y are both odd, prove that the product of the two is also odd.
2) For any odd integer x, prove that x^3 is odd
3) Prove that if n is a positive integer then $3^{2n} - 1$ is divisible by 8.

(hint: multiply out $(3^n - 1)(3^n + 1)$)

Trigonometric Identities

- Prove that $\sin^2 A + \cos^2 A = 1$



Forms of Proof

- Proof by Direct Argument
- Proof by Contradiction
- Proof by Exhaustion
- Proof by Induction

An Example of Proof by Contradiction

- Prove that $\sqrt{2}$ is irrational

Let us suppose that it is rational, i.e. that $\sqrt{2} = \frac{m}{n}$, where m and n have no common factors.

$$2 = \frac{m^2}{n^2}\text{, so } 2n^2 = m^2$$

m^2 is even, so m must also be even i.e. m = 2p

$$2n^2 = (2\text{p})^2 = 4\text{p}^2$$

$n^2 = 2p^2$, so n must also be even

However, this violates the assertion that m and n have no common factors, as they have a common factor of 2.

Therefore $\sqrt{2}$ is not rational i.e. irrational.

Try and apply that to the following problem!

Prove that $\log_2 3$ is irrational.